Two-bit Deutsch-Jozsa algorithm using an atomic ensemble

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Abstract.   The first optical proposal for the realization of the two-bit version of the Deutsch-Jozsa algorithm [D. Deutsch and R. Jozsa, Proc. R. Soc. London A **493**, 553 (1992)] is presented. The proposal uses Stark shifts in an ensemble of atoms and degenerate sources of photons. The photons interact dispersively with an atomic ensemble, leading to an effective Hamiltonian in atom-field basis, which is useful for performing the required two-qubit operations. Combining these with a set of one-qubit operations, the algorithm can be implemented. A discussion of the experimental feasibility of the proposal is given.

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It is known that the quantum computers are able to perform certain specific jobs like factorization, searching a database etc. much faster than its classical counterparts. Several quantum algorithms have been developed to demonstrate the power of quantum computers. For example, Grover has proposed an algorithm to search a quantum state from an unsorted database of 2^n states [1]. This requires $O(2^{n/2})$ repetitions of certain unitary operations, demonstrating quadratic speed-up than the classical computers. Deutsch-Jozsa algorithm [2] can be used to identify a certain function of n binary variables as constant or balanced by a single enquiry, whereas classically it would require up to $2^{n-1} + 1$ enquiries. The key of this enormous speed of quantum computers lies in the so-called quantum parallelism, which enables one to apply the same unitary operation simultaneously on a number of basis states.

There have been several proposals and experiments on implementation of these algorithms. For example, Grover's algorithm has been implemented using nuclear magnetic resonance (NMR) in bulk systems [3]. NMR [4, 5], ion trap [6], and linear optical [7] techniques have been employed to implement the Deutsch-Jozsa algorithm. Recently, an optical scheme to realize the one-bit version of the Deutsch-Jozsa algorithm was proposed [8]. To date, most of the works on Deutsch-Jozsa algorithm is for the case of a single qubit and we need to realize this algorithm for larger number of qubits, so that the full power of this algorithm can be realized. In this paper, we show how this can be achieved using Stark shifts in an ensemble of atoms. In this model, two freely propagating photonic qubits encoded in their polarization modes interact dispersively with an atomic ensemble. This leads to an effective Hamiltonian which along with several one-qubit operations enables us to implement the Deutsch-Jozsa algorithm for two qubits. We further discuss the experimental feasibility of our scheme using current technology.

We start with the main features of the algorithm. This algorithm helps to identify whether a bivalued function $f_i(x)$ (which can take only the values 0 or 1) of a variable x is constant or balanced. Here x is the decimal equivalent of the bit string $(x_1, x_2, ..., x_n)$ for n bits, i.e., $x = \sum_{i=1}^{n} x_i 2^{n-i}$. Thus, x can take any of the 2^n values between 0 and $2^n - 1$, for different binary combinations. If for half the values of x, the function takes the value 0 and for the other half, it takes the value 1, then the function is called balanced. The function will be called constant if it assumes either the value 0 or 1 for all values of x. In classical sense, it is easy to verify that at most $2^{n-1} + 1$ evaluations of the function $f_i(x)$ for different values of x are necessary to determine whether the function is constant or balanced [see Table 1]. The Deutsch-Jozsa algorithm, on the contrary, requires only a single "evaluation" of the function for the same. To demonstrate such an immense power of this algorithm, we show in Fig. 1 the basic circuit [9] to perform the n-bit version of the Deutsch-Jozsa algorithm. It can be shown that if all the qubits of the system A are in $|0\rangle$ state, then the function $f_i(x)$ is a constant function. But if at least one of these n qubits is in the state $|1\rangle$, then the function can be identified as a balanced function. In what follows, we will demonstrate the two-bit version of the Deutsch-Jozsa algorithm in an ensemble of atoms interacting with a quantized field.

Table 1. Different possible functions showing their characterizations for $n = 2$. Here x is the decimal equivalent of the bit string (x_1, x_2), e.g., $x = 2$ for two-bit input $(1,0)$.

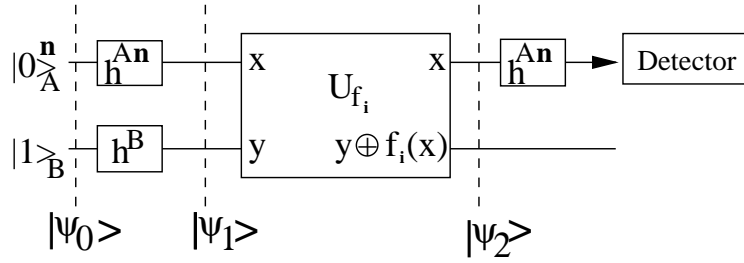| Input	Constant		Balanced					
x	$f_1(x)$	$f_2(x)$	$f_3(x)$	$f_4(x)$	$f_5(x)$	$f_6(x)$	$f_7(x)$	$f_8(x)$
0	0	1	0	1	0	1	0	1
1	0	1	0	1	1	0	1	0
2	0	1	1	0	0	1	1	0
3	0	1	1	0	1	0	0	1

Figure 1. Basic circuit to perform the Deutsch-Jozsa algorithm for n qubits. All the n qubits in the system A are initialized to the state $|0\rangle$, while the qubit in the ancillary system B is initialized to the state $|1\rangle$. h^A, h^B - Hadamard transformations of the qubits of the systems A and B, U_{f_i} - unitary operation operating on the system A+B, $|\psi_0\rangle$ etc. - states of the system A+B, $x \in \{0, \ldots, 2^n - 1\}$, $y \in \{0, 1\}$. The detector measures states of each qubit of the system A in $(|0\rangle, |1\rangle)$ basis.
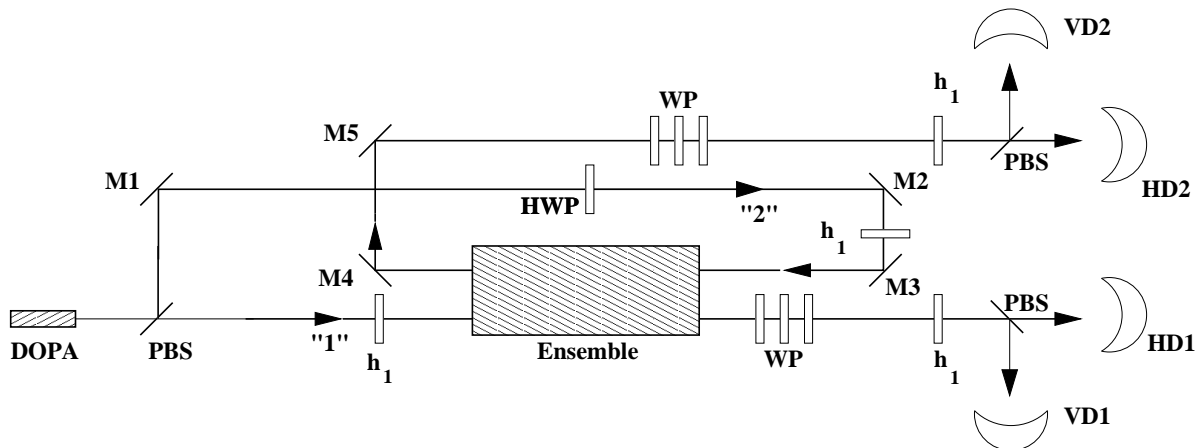

Figure 2. A possible experimental set-up for the two-bit Deutsch-Jozsa algorithm. DOPA - degenerate optical parametric amplifier, PBS - polarization beam splitter, M1 etc. - lossless mirrors, HD1, HD2, VD1, VD2 - single-photon detectors for horizontal (H) and vertical (V) polarizations, HWP - half-wave plate, h_1 - sequence of quarter-wave and half-wave plates to perform the operation h_1 for photons, WP - sequence of quarter-wave and half-wave plates to perform the operations h' and h'' for photons. The Hadamard rotations for the atomic ensemble are not shown.
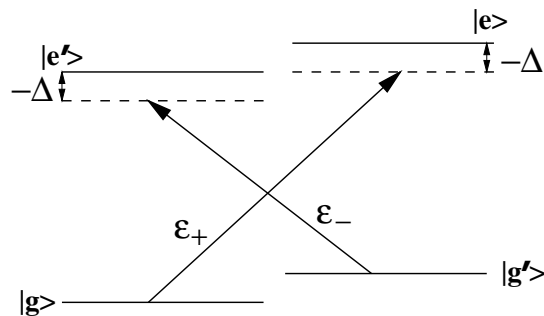
Figure 3. Relevant level configuration for implementing the Deutsch-Jozsa algorithm. The excited levels $|e\rangle$ and $|e'\rangle$ are coupled to the ground levels $|g\rangle$ and $|g'\rangle$, respectively, by the orthogonal polarization modes ϵ_+ and ϵ_- of the photons, with equal detuning $-\Delta$.

In order to implement the Deutsch-Jozsa algorithm for two qubits, we need two distinguishable photonic qubits. Each photonic qubit is of course formed from two states of polarization. We would thus use two single photons propagating in opposite directions so as to keep the qubits distinguishable. Note that single photons have been used extensively as qubits for quantum computation based on linear optics [10, 11]. Further, the implementation of the algorithm requires a number of Hadamard transformations of photons which can be done by using quarter-wave and half-wave plates. We show in Fig. 2 a possible experimental set-up for the interaction of distinguishable photonic qubits. We use a degenerate optical parametric amplifier to produce two photons simultaneously [12, 13, 14]. These photons interact with an ensemble of atoms. The experimental scheme shows how to make the photons distinct. The Deutsch-Jozsa algorithm requires that the polarization of two photons are measured simultaneously. For this purpose, we propose use of single-photon detectors [15]. The photon "1" ("2") can be detected in either of the detectors HD1 (HD2) and VD1 (VD2), which detects it in either horizontal or vertical polarization. The polarization beam splitter before the detectors preselects the polarization. Clearly, if the two detectors (HD1 or VD1 and HD2 or VD2) click simultaneously, the polarization of photons can be measured and thus the experiment leads to a conclusion for the algorithm.

We consider the interaction of two distinct photons with an ensemble of N atoms where only four levels are relevant as shown in Fig. 3. Note that this kind of configuration can be found in the clock transitions in ^{133}Cs atoms and has been studied very extensively in [16, 17]. Each photon has two circular polarization modes $\epsilon_\pm$. We designate these modes as two orthogonal states $|1,0\rangle_k$ and $|0,1\rangle_k$, for the kth photon, where $|1,0\rangle$ ($|0,1\rangle$) refers to a single photon in ϵ_+ (ϵ_-) mode and no photon in ϵ_- (ϵ_+) mode. The photons in the states $|1,0\rangle$ and $|0,1\rangle$ interact respectively with the $|e\rangle \leftrightarrow |g\rangle$ and $|e'\rangle \leftrightarrow |g'\rangle$ transitions of each atom. We assume that the common frequency difference Δ between the photon polarization mode and atomic transition is much larger than the atom-photon coupling constant g, which is the same for both the modes. In this regime, the ground states $|g\rangle$ and $|g'\rangle$ of each atom get Stark-

shifted. Using second-order perturbation theory, the respective shifts can be calculated as $\hbar|g|^2 n_+/\Delta$ and $\hbar|g|^2 n_-/\Delta$, where $n_\pm$ are the total number of photons in $\epsilon_\pm$ modes. As the excited states $|e\rangle$ and $|e'\rangle$ of the atoms remain unpopulated by interaction with the photons in large detuning regime, the effective Hamiltonian can be written as

$$H_{\text{eff}} = \hbar\lambda \sum_{j=1}^{N}\sum_{k=1}^{2} \left[|g_j\rangle\langle g_j||1,0\rangle_k\langle 1,0| + |g_j'\rangle\langle g_j'||0,1\rangle_k\langle 0,1|\right] \, , \quad \lambda = |g|^2/\Delta \,. \tag{1}$$

We rewrite the above Hamiltonian in terms of the horizontal ($|0\rangle_k$)and vertical ($|1\rangle_k$) polarization states of each kth photon as

$$H_{\text{eff}} = \frac{1}{2}\hbar\lambda \sum_{j=1}^{N}\sum_{k=1}^{2} [|g_j\rangle\langle g_j|(|0\rangle_k\langle 0| - |1\rangle_k\langle 1| + i|0\rangle_k\langle 1| + i|1\rangle_k\langle 0|)$$
$$+ \ |g_j'\rangle\langle g_j'|(|0\rangle_k\langle 0| - |1\rangle_k\langle 1| - i|0\rangle_k\langle 1| - i|1\rangle_k\langle 0|)] \, , \tag{2}$$

where $|0\rangle_k = \frac{1}{\sqrt{2}}(|1,0\rangle_k + |0,1\rangle_k)$ and $|1\rangle_k = \frac{1}{i\sqrt{2}}(|1,0\rangle_k - |0,1\rangle_k)$ are identified as two basis states of the kth photonic qubit. As we deal with only two states of each atom, the ensemble of N atoms can have 2^N states. We use two of them as the states of the atomic qubit, namely $\prod_{j=1}^{N}|g\rangle_j (\equiv |0\rangle_A)$ and $\prod_{j=1}^{N}|g'\rangle_j (\equiv |1\rangle_A)$. Note that as the atomic qubit consists of only the ground states of the atoms, decoherence due to spontaneous emission does not affect the process. Further, as the photons in free space can have long decay time, they do not lead to any effective decoherence as well.

Under the action of the Hamiltonian (2), the following evolution occurs:

$$|0\rangle_A|0,0\rangle_F \longrightarrow \ - e^{-i\lambda Nt}|0\rangle_A(-i|0\rangle + |1\rangle)^{\otimes 2} \, ,$$
$$|0\rangle_A|1,1\rangle_F \longrightarrow e^{-i\lambda Nt}|0\rangle_A(-i|0\rangle + |1\rangle)^{\otimes 2} \, ,$$
$$|0\rangle_A|0,1\rangle_F \longrightarrow \ - ie^{-i\lambda Nt}|0\rangle_A(-i|0\rangle + |1\rangle)^{\otimes 2} \, ,$$
$$|0\rangle_A|1,0\rangle_F \longrightarrow \ - ie^{-i\lambda Nt}|0\rangle_A(-i|0\rangle + |1\rangle)^{\otimes 2} \, , \tag{3}$$

where the state $|0,1\rangle_F$ represents the photon 1 in the horizontal polarization state $|0\rangle_F$ and the photon 2 in the vertical polarization state $|1\rangle_F$ and so on. Similar evolution occurs when the atomic qubit is initially in the state $|1\rangle_A$. Note that the Hamiltonian (2) changes the polarization of each photon from linear to circular polarization, i.e., the atomic ensemble acts as a polarizer for each photon. We next show how this change in polarization helps us to implement the Deutsch-Jozsa algorithm for two qubits. Now onwards, we choose the interaction time T of the atomic ensemble with the photons such that $\lambda NT = \pi/2$. As shown previously [8], this interaction time is achievable with the available technology [16, 17]. For example, in an ensemble of ^{133}Cs atoms in a cell of dimensions $10{\times}10{\times}200$ μm^3 at room temperature, the coupling constant g of each photon with the ensemble becomes 2.91×10^8 s^{-1} and time T of interaction of the photon with the ensemble becomes 6.67×10^{-13} s. Then for an atomic density of the order of 5×10^{12} cm^{-3} (i.e., for $N = 10^5$), the required detuning of each photon can be calculated from the above relation as $\Delta = 3.59$ GHz $\approx 12.35g$. On the other hand, for D_1 transitions ($5^2S_{1/2} \leftrightarrow 5^2P_{1/2}$, transition wavelength ~ 795 nm) in an ensemble of ^{87}Rb atoms in a magneto-optical trap (with diameter ~ 0.5 mm) [18], we calculate

$g \sim 3.53 \times 10^6$ s^{-1} and $T \sim 1.67 \times 10^{-12}$ s. Thus for $N \sim 2.5 \times 10^6$ in the trap, we find $\Delta \sim 9g$ Clearly, the condition $\Delta \gg g$ is well satisfied in both cases.

We now introduce the following four inequivalent Hadamard operations for each photon in $(|0\rangle_k, |1\rangle_k)$ basis and for each atom in the ensemble in $(|g\rangle_j, |g'\rangle_j)$ basis:

$$h_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \; ; h_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \; ;$$

$$h_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \; ; h_4 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} \; . \tag{4}$$

These operations can be implemented by using a resonant microwave field coupling the levels $|g\rangle_j$ and $|g'\rangle_j$ of the jth atom. The corresponding interaction Hamiltonian is given by

$$H_\mu^j = -\hbar\Omega \left[e^{i\phi} |g'\rangle_j \langle g| + \text{H.c.} \right] , \tag{5}$$

where Ω is the coupling constant of the microwave field with each atom in the ensemble and ϕ is the phase of the field with respect to the matrix element between the levels $|g\rangle_j$ and $|g'\rangle_j$. Under the action of this Hamiltonian, the rotations (4) for each atom can be obtained by choosing $\Omega t = \pi/4$ and $\phi = -\pi/2, 0, \pi/2, \pi$, respectively. Note that we do not provide the above rotations for the collective atomic states $|0\rangle_A$ and $|1\rangle_A$. Alternatively, one could use two classical fields in Raman resonance to provide the equivalent rotation for each atom in the ensemble. For the photonic qubits, it is easy to verify that the operations (4) refer to rotation of linear polarizations to linear or circular polarizations, which can be performed using different sequences of quarter-wave and half-wave plates. Note that the matrix forms for transformations through quarter-wave plates and half-wave plates are given by

$$Q_\phi = \frac{i}{\sqrt{2}} \begin{pmatrix} \cos(2\phi) - i & \sin(2\phi) \\ \sin(2\phi) & -\cos(2\phi) - i \end{pmatrix} \; ;$$

$$H_\phi = i \begin{pmatrix} \cos(2\phi) & \sin(2\phi) \\ \sin(2\phi) & -\cos(2\phi) \end{pmatrix} , \tag{6}$$

in $(|0\rangle_k, |1\rangle_k)$ basis, where ϕ is the angle of alignment of the wave-plate with the axis perpendicular with its plane. The single-qubit operations (4) for photons can be obtained using a series of these wave-plates of different orientations ["the SU(2) gadget"] as follows [19, 20]:

$$h_1 \equiv Q_{\pi/4} Q_{\pi/4} H_{-3\pi/8} \; ; \quad h_2 \equiv Q_{\pi/4}$$

$$h_3 \equiv Q_{\pi/4} Q_{\pi/4} H_{-\pi/8} \; ; \quad h_4 \equiv Q_{-\pi/4} . \tag{7}$$

Note that the half-wave plate rotates the polarization state from $|0\rangle_k$ to $|1\rangle_k$ and vice versa for $\phi = \pi/4$.

Further, as required by the Deutsch-Jozsa algorithm, two photonic qubits should be initially in the same polarization state $|0\rangle_k$. This can be done by using a half-wave plate [see Fig. 2]. The degenerate parametric amplifier produces two photons in horizontal and vertical polarization states. One of them ("2") is in vertical polarization $|1\rangle$ and

thus is reflected by a polarization beam splitter. This photon is then sent through a half-wave plate (HWP in Fig. 2) that rotates its polarization from $|1\rangle$ to $|0\rangle$, while the other photon ("1"), which is already in horizontal polarization state $|0\rangle$, passes through the polarization beam splitter. Thus both the photons are initialized to the state $|0\rangle_k$.

We now discuss how one can implement the Deutsch-Jozsa algorithm for two qubits using the evolution (3) and (4). We assume that two photons (the atomic ensemble) serve the purpose of the system A (B) in Fig. 1. We start with the state $|\psi_0\rangle_{F+A} = |0,0\rangle_F|1\rangle_A$ of the photons+ensemble system. Applying the Hadamard rotation h_1 on each photon and the atomic qubit, we obtain

$$|\psi_1\rangle_{F+A} = \left[\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right]_F^{\otimes 2} \prod_{j=1}^{N} \frac{1}{\sqrt{2}}(|g_j\rangle - |g_j'\rangle) . \tag{8}$$

Next the U_{f_i} operations are applied. We show that in the present case, $U_{f_i} = h_i^{\mathrm{eq}} U_{\mathrm{eff}} h_1^A$, where $U_{\mathrm{eff}} = \exp[-iH_{\mathrm{eff}}t]$ is the unitary operator corresponding to the Hamiltonian (2). Here the operations h_i^{eq} correspond to different functions $f_i(x)$.

In course of the operator sequence U_{f_i}, the operation h_1^A first prepares the atoms in $|0\rangle_A$ state:

$$|\psi_1'\rangle_{F+A} = \left[\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right]_F^{\otimes 2} |0\rangle_A . \tag{9}$$

Then the operation U_{eff} for an interaction time T defined by $\lambda NT = \pi/2$ yields the following [see Eqs. (3)]:

$$|\psi_1''\rangle_{F+A} \equiv \left[\frac{1}{\sqrt{2}}(-i|0\rangle + |1\rangle)\right]_F^{\otimes 2} |0\rangle_A . \tag{10}$$

We next identify the operations h_i^{eq} for different possible balanced functions $f_i(x)$ [see Table 1] as follows:

$$h_{3,4}^{\mathrm{eq}} \equiv h_{F_1}'' h_{F_2}' , \quad h_{5,6}^{\mathrm{eq}} \equiv h_{F_1}' h_{F_2}'' , \quad h_{7,8}^{\mathrm{eq}} \equiv h_{F_1}'' h_{F_2}'' . \tag{11}$$

Here $h' = h_1 h_4 h_3$ and $h'' = h_1 h_2 h_3$ are composite sequences of different Hadamard rotations defined in Eq. (4) and yield the following:

$$h'|0\rangle = e^{i\pi/4}|0\rangle , \ h'|1\rangle = e^{-i\pi/4}|1\rangle ;$$
$$h''|0\rangle = e^{-i\pi/4}|0\rangle , \ h''|1\rangle = e^{i\pi/4}|1\rangle . \tag{12}$$

Note that h_{F_i}' refers to h' operations on ith photon. The operators h_i^{eq} lead to the following results, when applied on the state $|\psi_1''\rangle$ in Eq. (10):

$$
\begin{aligned}
|\psi_2\rangle_{f_3,f_4} &= h_{3,4}^{\mathrm{eq}}|\psi_1''\rangle \\
&= \pm\frac{1}{2}(|0,0\rangle + |0,1\rangle - |1,0\rangle - |1,1\rangle)_F|0\rangle_A \\
&= \pm\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)_{F_1}\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_{F_2}|0\rangle_A \\
|\psi_2\rangle_{f_5,f_6} &= h_{5,6}^{\mathrm{eq}}|\psi_1''\rangle \\
&= \pm\frac{1}{2}(|0,0\rangle - |0,1\rangle + |1,0\rangle - |1,1\rangle)_F|0\rangle_A
\end{aligned}
\tag{13}
$$

$$= \pm \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_{F_1} \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)_{F_2}|0\rangle_A \qquad (14)$$

$$|\psi_2\rangle_{f_7,f_8} = h_{7,8}^{\text{eq}}|\psi_1''\rangle$$

$$= \pm \frac{1}{2}(|0,0\rangle - |0,1\rangle - |1,0\rangle + |1,1\rangle)_F|0\rangle_A$$

$$= \pm \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)_{F_1} \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)_{F_2}|0\rangle_A . \qquad (15)$$

The final step of the algorithm is to apply the Hadamard rotation h_1^F on each photon, which leads to the following outcomes:$|\psi_3\rangle_{f_3,f_4} \equiv |0,1\rangle_F|0\rangle_A$, $|\psi_3\rangle_{f_5,f_6} \equiv |1,0\rangle_F|0\rangle_A$, and $|\psi_3\rangle_{f_7,f_8} \equiv |0,0\rangle_F|0\rangle_A$. Thus, after the final Hadamard rotations, either at least one of the photons remains in $|0\rangle_F$ state, if the functions were balanced.

On the other hand, in case of constant function f_1, the U_{f_1} operation is equivalent to the identity operation, while for the function f_2, the U_{f_2} operation is equivalent to the NOT operation on the atomic qubit. This can be implemented using microwave field for $\Omega t = \pi/2$ and $\phi = \pi/2$ [see the Hamiltonian (5)]. Then, the output states $|\psi_3\rangle_{f_1,f_2}$ of the photons+ensemble system become $|1,1\rangle_F \prod_{j=1}^N \frac{1}{\sqrt{2}}(|g\rangle_j \pm |g'\rangle_j)$. This means that if the functions were constant, both the photons can be detected in the state $|1\rangle_F$. Clearly, by measuring the polarization states of two photons at the end of the algorithm, one can characterize whether the functions were constant or balanced.

To this end, we discuss possibility of decoherence in the entire process. As the photons interact with the ensemble dispersively, the excited states of the atoms get hardly populated. So the effect of spontaneous emission during the process is minimal. However, there could be collisional relaxation of the atomic ground states, which leads to loss of coherence in these states. But the time-scales of both these relaxation processes are of the order of 10^{-6} s, whereas, interaction time T of the photon with the ensemble is of the order of 10^{-13} s. Clearly, this is much less than the time at which the effects of different relaxation processes set in significantly. Thus, our model with photons and atomic ensemble is virtually decoherence-free.

In conclusion, we demonstrated how the Deutsch-Jozsa algorithm can be implemented for two qubits using an atomic ensemble. One requires two distinguishable photons for this. We show how this can be achieved by using the output of a degenerate parametric amplifier. These photons dispersively interact with an ensemble of N atoms. The resulting Stark shifts of the atomic states lead to an effective Hamiltonian, which along with several Hadamard transformations, are used to implement the algorithm. We provide all the relevant operations to implement the algorithm and discussed the expected outcomes.

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